# Holographic monopole catalysis of baryon decay

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ABSTRACT: We study how monopole catalysis of baryon decay is realized in holographic QCD. Physics of monopole catalysis becomes much simpler in holographic description as it occurs due to the violation of the Bianchi identity for the 5D gauge symmetry when magnetic monopole is present. In holographic QCD we find a unified picture of the baryon number violation under magnetic monopole or electroweak sphaleron, giving a new mechanism of baryon number violation. We also embed our set-up in the string theory model by Sakai and Sugimoto.

## Contents

## 1. Introduction

Recent development [1] of the dual correspondence between gauge theories and string theories has given us a powerful tool to investigate strongly coupled gauge theories like Quantum Chromodynamics (QCD) or Technicolor, which are otherwise extremely difficult to solve. According to the gauge/string duality the low energy QCD becomes a theory of mesons in the large number of color ($N_c$) and large 't Hooft coupling ($\lambda \equiv g_s^2 N_c$) limit, but in a warped five-dimensional spacetime. The theory, known as holographic QCD, is nothing but a 5D flavor gauge theory in the warped background geometry, endowed with a Chern-Simons term, necessary to realize the global anomalies of QCD [2, 3].

Being the theory of mesons in the large $N_c$ limit, holographic QCD should admit baryons as solition solutions, as conjectured by Skyrme long time ago for the nonlinear sigma model [4]. Indeed, it was found that the baryons are realized as instanton solitons in holographic QCD [5].[1] The instanton picture of baryons reproduces the success of skyrmions rather well but with much less parameters for the spectrum and the static properties of baryons [7 – 10]. Unlike skyrmions, however, the instanton solitons are made of not only pions but infinite towers of vector mesons, intertwined nontrivially, leading to small size objects without any intrinsic core, which therefore realizes full vector meson dominance for baryons [7, 10].

One of nice features of skyrmion is that monopole catalysis of baryon decay [11, 12] is easily described in the skyrmion picture of baryons, which then sets up the practical basis for the calculation of monopole catalysis [13]. The magnetic monopole provides a defect on which the topological charge of skyrmions can unwind, allowing skyrmions to decay at the rate of QCD scale without suppression by the monopole scale.

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[1]There is an alternative realization of baryons through 5D holographic baryon fields [6].

In this paper we describe how monopole catalysis of baryon decay realizes in holographic QCD. The magnetic monopole catalyzes baryon decay, since the 5D baryon number current, $B^M = (1/32\pi^2)\epsilon^{MNPQR}\mathrm{Tr}\,F_{NP}F_{QR}$ is not conserved in the presence of the holographic magnetic monopole string by violating the Bianchi identity for the gauge fields. Furthermore, we show that monopole catalysis can be naturally described in string theory as the dissolution of $D4$ brane (instanton soliton) into $D6$ brane (monopole string), which suggests that monopole catalysis of baryon decay should occur without any barrier and hold even beyond the large $N_c$ limit. The decay rate of baryons by monopole catalysis is determined by the scale of instanton solitons. In section 2 we first briefly review the monopole catalysis of skyrmion decay, studied by Callan and Witten [13], and then in sections 3 and 4 we describe how monopole catalysis is realized in holographic QCD. Finally in section 5 we present the string theory realization of monopole catalysis of baryon decay. Section 6 contains concluding remarks together with future directions.

## 2. Monopole catalysis of baryon decay

Monopoles can arise in grand unified theories (GUTs) as the unified gauge symmetry breaks down to the Standard Model gauge group. Their typical size is about the unification scale, which is nearly point-like compared to the usual scales of the Standard model physics, especially QCD. Since electromagnetism is the only long-ranged interaction in the Standard Model, a GUT monopole eventually looks like an electromagnetic Dirac monopole to low energy observers. Near the core of the monopole, there are clouds of heavy GUT gauge fields which can mediate various GUT interactions, among which are baryon number violating processes. Despite its small size, the cross sections for these monopole-induced processes are not suppressed by the unification scale, due to their origin being related to chiral anomalies. Rather, the monopole center provides a baryon number violating vertex of unit strength acting as a catalysis of baryon decay, and the actual cross section is governed by low energy dynamics such as QCD.

The physics gets more interesting when the low energy dynamics such as QCD becomes strongly coupled and takes a completely different looking effective theory. Because the monopole catalysis is not suppressed by any scales of the UV theory, its presence must persist even in the low energy effective theory, and we need a non-trivial disguise of the monopole-induced baryon decay as an interesting low energy phenomenon within the effective theory. As the structure of the monopole core given by the UV physics is not a concern of the low energy effective theory, the Dirac monopole profile of the unbroken gauge group should be sufficient for the low energy description of monopole catalysis. This indicates an intricate theoretical consistency requirement for the physics induced by Dirac monopoles in any low energy effective theory.

A consistent low energy effective theory of QCD is the chiral Lagrangian of $\mathrm{SU}(N_F)_L \times \mathrm{SU}(N_F)_R$, and the low energy baryons are effectively described by topological solitons, called Skyrmions, in the large $N_c$ limit. Assuming that the large $N_c$ QCD can be embedded in some GUT theory which contains monopoles capable of baryon number catalysis, the above discussion leads to the expectation that an electromagnetic Dirac monopole should be

able to induce Skyrmion-baryon decay within the low energy chiral dynamics. At first sight, this looks puzzling because the baryon number of the Skyrmions is purely topological and there seems to be no way for the Skyrmions to decay within the framework of the effective theory. This is the problem of monopole catalysis of Skyrmion decay analyzed by Callan and Witten long ago [13].

The resolution of the puzzle is that the baryon number and its current must be modified in the presence of a background electromagnetic field,[2] to be gauge invariant and conserved at the same time. This requirement determines the baryon number current uniquely [13]. More explicitly, the standard baryon number current of Skyrmions which is topologically conserved is given by

$$B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\text{Tr}\left(U^{-1}\partial_\nu U U^{-1}\partial_\alpha U U^{-1}\partial_\beta U\right) \quad , \tag{2.1}$$

where $U$ is the SU(2) group field of the chiral Lagrangian[3] which transforms as

$$U \to g_L U g_R^\dagger \quad , \tag{2.2}$$

under the chiral symmetry $\text{SU}(2)_L \times \text{SU}(2)_R$. Since the electromagnetic $\text{U}(1)_{EM}$ acts on $U$ by

$$U \to e^{iQ}Ue^{-iQ} \quad , \quad Q = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} \quad , \tag{2.3}$$

the simplest way to make the baryon current gauge invariant would be to replace the ordinary derivatives $\partial U$ by the covariant derivatives $DU = \partial U + A^{EM}[Q,U]$, where $A^{EM}$ is the electromagnetic gauge potential.[4] However, the resulting baryon current is no longer conserved in general, and we need to add more gauge-invariant terms to make it conserved. This has been worked out in ref. [13], and we quote the result

$$\begin{aligned} B^\mu &= \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\text{Tr}\left(U^{-1}\partial_\nu U U^{-1}\partial_\alpha U U^{-1}\partial_\beta U\right) \\ &\quad - \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\partial_\nu\left[3A_\alpha^{EM}\text{Tr}\left(Q(U^{-1}\partial_\beta U + \partial_\beta U U^{-1})\right)\right] \quad . \end{aligned} \tag{2.4}$$

It is clear that the final form of the modification does not affect the conservation of the current, while it can also be checked that the result is $\text{U}(1)_{EM}$ gauge invariant.

However, the conservation of baryon number, $\partial_\mu B^\mu = 0$, is guaranteed only for a smooth, well-defined background potential $A^{EM}$. Since a Dirac monopole field is singular and doesn't have a well-defined potential, its presence might invalidate the conservation of the above gauge invariant baryon number. Indeed, this is exactly what causes Skyrmions to decay in the presence of a monopole. One might question that the topological Skyrmion number (2.1) can never be violated under a smooth time evolution, and

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[2]In general, any background $\text{SU}(N_F)_L \times \text{SU}(N_F)_R \times \text{U}(1)_B$ gauge fields upon weakly gauging it requires modification of the baryon number current.

[3]For simplicity, we will confine our discussion to the $N_F = 2$ massless quarks

[4]We use the convention where the gauge potential is *anti*-hermitian, and the covariant derivative is $D = \partial + A$. To compare with ref. [13], simply replace $A$ by $-ieA$.

an initial Skyrmion would never decay to topologically trivial meson states. However, a caveat is that a Dirac monopole entails the Dirac-string on which $A^{EM}$ is singular, and we are allowed to take singular gauge transformations to move this string from one direction to another. Under these singular $U(1)_{EM}$ gauge transformations which are now allowed in a Dirac monopole background, the topological Skyrmion number can actually change. In other words, the Skyrmion number is not a well-defined gauge-invariant quantity in the presence of a Dirac monopole, and a configuration with non-zero Skyrmion number can be equivalently described by a topologically trivial configuration under a gauge transformation. What remains invariant under the gauge transformations is the new gauge-invariant baryon number (2.4).

This gauge-invariant baryon number can be eaten up at the monopole center dynamically, which is responsible for the baryon decay. Near the center of the monopole, only the neutral component of the pion, $\pi^0$, can take non-zero values, while charged pion excitations would cause too much energy, since they have nonzero angular momentum proportional to the magnetic charge of monopole. Writing $U(t) = \exp(\frac{2i}{F_\pi}\pi^0(t)\sigma^3)$ near the center of a Dirac monopole of unit strength

$$A^{EM} = -\frac{i}{2}(1 - \cos\theta)d\phi \quad , \tag{2.5}$$

and using $\epsilon^{r\theta\phi t} = \frac{-1}{\sqrt{g}} = \frac{-1}{r^2\sin\theta}$ in the polar coordinate, the radial flux of the baryon number out of the monopole is readily calculated to be

$$B^r = \frac{\partial_t \pi^0}{4\pi^2 F_\pi r^2} \quad , \tag{2.6}$$

whose integration gives the change of baryon number

$$\frac{dB}{dt} = \frac{1}{\pi F_\pi}(\partial_t \pi^0) \quad . \tag{2.7}$$

Therefore, the rate of change of $\pi^0$ at the monopole center is proportional to disappearance of the baryon charge from the effective theory. In the original GUT, this baryon number violation should be accompanied by creation of leptons, whose detail should resort to some unknown dynamics at the center of the monopole, so that the total fermion number is conserved. In the low energy effective theory, this normally involves putting a relevant boundary condition on the leptons at the monopole center, such that the change of baryon number is compensated by the change of lepton number. In the following sections, we will see that all the above features nicely fit into a simple description in the framework of holographic QCD.

## 3. Holographic baryon number current

To be specific, we present our analysis in the model by Sakai and Sugimoto (SS) from Type IIA string theory (See ref. [14 – 16] for its quark mass deformation). However, most of the steps we perform are dictated by symmetry and don't in fact depend on the details of the model, and hence our analysis is applicable to any model of holographic QCD.

In the SS model, the world volume $U(N_F)_L$ and $U(N_F)_R$ gauge fields on $D8$ and $\bar{D}8$ in UV region are holographically dual to the corresponding chiral symmetry in QCD. Its spontaneous breaking to the diagonal $U(N_F)_I$ is geometrically realized by adjoining $D8$ and $\bar{D}8$ at the tip of the cigar geometry. Alternatively, we can view this as having a single $D8$ brane whose two asymptotic boundaries towards UV region encode the chiral symmetry $U(N_F)_L$ and $U(N_F)_R$ respectively. The latter view point is more practical in the analysis, and we introduce a coordinate $z$ on $D8$ such that $z \to \pm\infty$ represent two UV boundaries. We will call $z$ the radial or the 5th direction. Assuming homogeneity along the internal $S^4$ fibration, the world volume theory on $N_F$ $D8$ branes is effectively a 5D $U(N_F)$ gauge theory in a non-trivial $z$ dependent background. According to AdS/CFT correspondence, the asymptotic values of the 5D gauge potential near the two boundaries, $A_\mu(x, z \to \infty)$ and $A_\mu(x, z \to -\infty)$,[5] are non-dynamical background fields coupled to QCD $U(N_F)_L$ and $U(N_F)_R$ currents respectively. Equivalently, they are precisely the background gauge potential upon weakly gauging the chiral symmetry.

Note that the above prescription holds true in the gauge where $A_z$ is kept free (and vanishes at $z \to \pm\infty$), while it is often more convenient to work in the gauge where $A_z = 0$. After performing a suitable gauge transformation from the above to the $A_z = 0$ gauge, the boundary behavior of $A_\mu$ will be slightly different from the above. However, any gauge invariant calculations are independent of the gauge choice.

The 5D gauge theory on $D8$ also contains a tower of normalizable (axial) vector meson excitations in view of 4D observers. Especially, the Wilson line

$$U(x^\mu) = P \exp\left( - \int_{-\infty}^{+\infty} dz\, A_z(x^\mu, z) \right) \quad , \tag{3.1}$$

is identified as the massless Nambu-Goldstone pion for the chiral symmetry breaking $U(N_F)_L \times U(N_F)_R \to U(N_F)_I$,[6] and it is precisely the group field entering the low energy QCD chiral Lagrangian. Upon expanding $A_M$ in terms of normalizable (axial) vector mesons as well as the non-normalizable background fields previously mentioned,[7] and performing $z$-integration we obtain a 4D effective chiral Lagrangian of $U(x)$ and excited mesons coupled to the background gauge potential. The part that contains $U(x)$ reproduces the previously known gauged Skyrmion theory with the correct Wess-Zumino-Witten term. Therefore, the 5D gauge field compactly summarizes the pions, the excited mesons, and the background gauge potential of the chiral symmetry in a single unified framework.

As the 5D gauge theory on $D8$ branes includes the Skyrmion theory, there must exist topological objects similar to Skyrmions that play a role of baryons. Indeed, the 5D gauge theory has topological solitons whose field profile on the spatial $(x^\mu, z)$ directions taking that of instantons. We call them instanton-baryons not to be confused with real

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[5] We use Greek indices for the Minkowki $R^{1,3}$ directions, while the full 5D coordinates $(x^\mu, z)$ will be denoted by capital letters.

[6] Note that axial anomaly of $U(1)_A$ is negligible in the large $N_c$ limit.

[7] The modes from $A_z$, except the Wilson line (3.1), are eaten after all by massive spin 1 (axial) vector mesons coming from $A_\mu$. In this sense, the $A_z = 0$ gauge is a kind of unitary gauge where only physical degrees of freedom are present.

instantons in Euclidean field theory. This also agrees with the baryonic objects coming from $S^4$-wrapped $D4$ branes in the string theory of this background, since these $D4$ branes can dissolve into $D8$ branes exactly as instanton-solitons [17]. The topology of instanton-baryons is counted by the instanton number

$$B = \frac{1}{32\pi^2} \int dz dx^3 \epsilon^{MNPQ} \text{Tr} \left( F_{MN} F_{PQ} \right) = \frac{1}{8\pi^2} \int_{R^4} \text{Tr} \left( F \wedge F \right) \quad, \tag{3.2}$$

where $M, N, P, Q$ spans only spatial dimensions and the epsilon tensor is defined in the flat space. In our convention, $F = dA + A \wedge A$. Being topological, its conservation is guaranteed in any smooth situations.

We can easily find the corresponding conserved current in 5D [5]. From the Bianchi identity $DF \equiv dF + A \wedge F - F \wedge A = 0$, we have

$$d\text{Tr} \left( F \wedge F \right) = \text{Tr} \left( DF \wedge F \right) + \text{Tr} \left( DF \wedge F \right) = 0 \quad, \tag{3.3}$$

and the current 1-form defined by $j_B = *_5 \text{Tr} \left( F \wedge F \right)$ is conserved

$$d *_5 j_B = 0 \quad. \tag{3.4}$$

Note that this is independent of what metric we use in defining $*_5$ since the conservation is a consequence of the Bianchi identity. In fact, the results of the following discussions will not be affected by metric at all, and for simplicity we will keep the flat 5D metric in $(x^\mu, z)$ coordinate whenever we need the metric as intermediate steps. In components (3.4) is written as $D_M j_B^M = \partial_M j_B^M = 0$, where $j_B^M = g^{MN} j_{BN} = \eta^{MN} j_{BN}$ and $D_M$ is the metric covariant derivative. This means that we can define a 4D conserved current $B^\mu$ by simply integrating $j_B^\mu$ along the $z$-direction,

$$B^\mu \equiv \int_{-\infty}^{+\infty} dz\, j_B^\mu \quad, \tag{3.5}$$

where the conservation is shown by

$$\partial_\mu B^\mu = \int_{-\infty}^{+\infty} dz\, \partial_\mu j_B^\mu = -\int_{-\infty}^{+\infty} dz\, \partial_5 j_B^5 = j_B^5(-\infty) - j_B^5(+\infty) = 0 \quad, \tag{3.6}$$

as the boundary term

$$j_B^5(\pm\infty) \sim \epsilon^{\mu\nu\alpha\beta} \text{Tr} \left( F_{\mu\nu} F_{\alpha\beta} \right) |_{z \to \pm\infty} \tag{3.7}$$

is the chiral sphaleron density of the background gauge potential of the chiral symmetry, and we assume that it vanishes for now. This is justified in our consideration of static monopoles without electric fields which will be discussed in a moment. We will come back to the implication of these boundary terms later. The explicit form of the 4D conserved baryon current $B^\mu$ is

$$B^\mu = \int_{-\infty}^{+\infty} dz\, j_B^\mu \sim \int_{-\infty}^{+\infty} dz\, \epsilon^{\mu\nu\alpha\beta} \text{Tr} \left( F \wedge F \right)_{\nu\alpha\beta z} = \eta^{\mu\nu} \left[ *_4 \int_z \text{Tr} \left( F \wedge F \right) \right]_\nu \quad. \tag{3.8}$$

The final form makes it clear that the end result is indeed metric-independent. By comparing with the normalized instanton number (3.2) as $\int d^3x B^0$, we can easily fix the normalization to be

$$B^\mu = \frac{1}{8\pi^2} \int_{-\infty}^{+\infty} dz \, \epsilon^{\mu\nu\alpha\beta} \, \text{Tr} \left( F_{\nu\alpha} F_{\beta z} \right) \quad . \tag{3.9}$$

Upon expanding $A_M$ in terms of the group field $U(x)$ (as well as excited mesons) and performing $z$-integration, we naturally expect that it reduces to the usual Skyrmion number current in (2.1).

A crucial point is that the above baryon current (3.9) remains gauge invariant even in the presence of background gauge potentials for the chiral symmetry, which are encoded as non-normalizable modes of $A_M$. Its conservation, relying on the Bianchi identity, is also intact in any smooth situations up the boundary term in (3.7). However, this boundary term cancels in (3.6) in the case of vector-like background field, that is, the fields coupled to $U(N_F)_I$ such that $A_\mu(+\infty) = A_\mu(-\infty)$. The electromagnetism belongs to this case with

$$A(+\infty) = A(-\infty) = QA^{EM} \quad , \quad Q = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} \quad . \tag{3.10}$$

Because these two constraints uniquely fix the baryon current (2.4) in the presence of background electromagnetic potential, the above holographic baryon current must reproduce (2.4) as its lowest component involving $U(x)$.

To check this, it is convenient to work in the $A_z = 0$ gauge with an expansion [2]

$$A_\mu(x,z) = A_{L\mu}^{\xi_+}(x)\psi_+(z) + A_{R\mu}^{\xi_-}(x)\psi_-(z) + \text{(excited modes)} \quad , \tag{3.11}$$

where

$$A_{L\mu}^{\xi_+} = \xi_+ (A_L)_\mu \xi_+^{-1} + \xi_+ \partial_\mu \xi_+^{-1} \quad , \quad A_{R\mu}^{\xi_-} = \xi_- (A_R)_\mu \xi_-^{-1} + \xi_- \partial_\mu \xi_-^{-1} \quad . \tag{3.12}$$

The group field $U(x)$ is contained in the above by $\xi_+^{-1}\xi_- = U$, and we denote the background gauge potential by $A_L = A(+\infty)$ and $A_R = A(-\infty)$. There still remains a residual gauge symmetry to fix, called Hidden Local Symmetry, which is nothing but the gauge transformation at the deepest IR $z = 0$ which acts on the partial Wilson lines $\xi_\pm$ as $\xi_\pm(x) \to h(x)\xi_\pm(x)$. For our purpose, we take the gauge $\xi_+^{-1} = U$ and $\xi_- = 1$, upon which we have

$$A_\mu = \left[ \left( U^{-1}QU \right) \psi_+ + Q\psi_- \right] A_\mu^{EM} + \psi_+ U^{-1}\partial_\mu U + \text{(excited modes)} \quad , \tag{3.13}$$

where we have explicitly used the electromagnetic background potential (3.10). The details of the zero mode wavefunctions $\psi_\pm(z)$ won't be important later, except $\psi_+ + \psi_- \equiv 1$ and $\psi_+(\infty) = \psi_-(-\infty) = 1$. From

$$\begin{aligned}
F_{\nu\alpha} &= \left( \left( U^{-1}QU - Q \right) \psi_+ + Q \right) F_{\nu\alpha}^{EM} - \psi_+(1 - \psi_+) \left[ U^{-1}\partial_\nu U, U^{-1}\partial_\alpha U \right] \\
&\quad + \psi_+(1 - \psi_+) \left( U^{-1}\partial_\nu U \left( Q - U^{-1}QU \right) + \left[ U^{-1}, Q \right] \partial_\nu U \right) A_\alpha^{EM} - (\nu \leftrightarrow \alpha) \quad , \\
F_{\beta z} &= -(\partial_z \psi_+) \left( \left( U^{-1}QU - Q \right) A_\beta^{EM} + U^{-1}\partial_\beta U \right) \quad ,
\end{aligned} \tag{3.14}$$

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and integrating over $z$ in (3.9), we can easily check that the result indeed agrees with the previously known 4D result (2.4). Observe that the $z$-integration involves only the following integrals

$$\int_{-\infty}^{+\infty} dz \, (\partial_z \psi_+)(\psi_+)^n = \frac{1}{n+1}(\psi_+)^{n+1}\Big|_{-\infty}^{+\infty} = \frac{1}{n+1} \quad , \qquad (3.15)$$

without regard to a detailed functional form of $\psi_+$. This can be understood because the final result is dictated by symmetry and it should be true universally for any holographic model of QCD.

In the presence of a general chiral background potential, we naturally propose (3.9) to be the right answer for the modified baryon current. With this granted, a violation of the baryon number by the boundary term (3.7)

$$\partial_\mu B^\mu \sim \epsilon^{\mu\nu\alpha\beta} \text{Tr}\,(F_{\mu\nu}F_{\alpha\beta})\Big|_R - \epsilon^{\mu\nu\alpha\beta} \text{Tr}\,(F_{\mu\nu}F_{\alpha\beta})\Big|_L \quad , \qquad (3.16)$$

implies that baryon number can be generated in an environment with non-zero chiral-asymmetric sphaleron density. This can be achieved by sphalerons made of elecro-weak gauge bosons, which are indeed known to induce baryon asymmetry via chiral anomaly. Our result can be thought of as a manifestation of this physics in the low energy effective theory.

For a reference, we obtain from (3.9) the baryon current in a general chiral background potential $A_L$ and $A_R$,

$$\begin{aligned}
B^\mu = {}& \frac{1}{24\pi^2}\, \epsilon^{\mu\nu\alpha\beta}\, \text{Tr}\,\left(U^{-1}\partial_\nu U U^{-1}\partial_\alpha U U^{-1}\partial_\beta U\right) \\
& -\frac{1}{8\pi^2}\, \epsilon^{\mu\nu\alpha\beta}\text{Tr}\,\partial_\nu\left(U^{-1}A_{L\alpha}\partial_\beta U + A_{R\alpha}U^{-1}\partial_\beta U - U^{-1}A_{L\alpha}U A_{R\beta}\right) \\
& -\frac{1}{8\pi^2}\, \epsilon^{\mu\nu\alpha\beta}\, \text{Tr}\,\left(\partial_\nu A_{L\alpha}\,A_{L\beta} + \frac{2}{3}A_{L\nu}A_{L\alpha}A_{L\beta} - (L \leftrightarrow R)\right) .
\end{aligned} \qquad (3.17)$$

This is an extension of (2.4), which we find via holographic QCD.

## 4. Holographic monopole catalysis of baryon decay

In this section, we re-analyze the monopole-induced baryon decay we discussed in section 2 in the framework of holographic QCD, and find that the physics becomes more transparent in holographic QCD.

A 4D background electromagnetic Dirac monopole enters our holographic model as a specific non-normalizable mode in the expansion of 5D gauge field $A_M$. In the gauge where we keep $A_z$ free, which is more convenient than the previous $A_z = 0$ gauge for the present purpose, the background potential appears in the expansion as

$$A_\mu(x,z) = \frac{1}{2}\left(A_L + A_R\right)_\mu + \frac{1}{2}\left(A_L - A_R\right)_\mu \psi_0(z) + \text{(normalizable modes)} \quad , \qquad (4.1)$$

where $\psi_0(z) = \frac{2}{\pi}\tan^{-1}(z)$ with $\psi_0(+\infty) = -\psi_0(-\infty) = 1$. $A_z$ contains only normalizable modes. For electromagnetism which is vector-like, $A_L = A_R = QA^{EM}$, the second term

is absent and we observe that a 4D monopole background would enter the 5D expansion homogeneously along the $z$-direction. In fact, a 5D gauge theory doesn't allow topological monopoles, but instead can have string-like objects (monopole-string) whose 3-dimensional transverse profiles resemble those of monopoles. Therefore, the natural holographic object corresponding to a 4D monopole is a monopole-string extending along the radial direction. Behaviors of normalizable modes, including the pions $\mathrm{U}(x)$, can be analyzed by studying the 5D gauge field fluctuations around the monopole-string background.

A nice thing in this holographic set-up is that the violation of baryon number (3.9) in the presence of a monopole-string has a simple explanation in terms of a violation of the Bianchi identity due to the magnetic source. The basic reason behind this simplification is that holographic QCD unifies dynamical degrees of freedom of the model, such as $\mathrm{U}(x)$, with the background potential $A^{EM}$ in a single 5D gauge theory framework, so that their physics should find its explanations within the 5D gauge theory. We should also point out that in the monopole-string background, the normalizable modes in the above expansion (4.1) will in general be excited by back-reactions, and the full field configuration is more complicated than the Dirac monopole alone. However, the amount of violation of the Bianchi identity doesn't depend on these meson clouds due to its topological nature, and is localized at the core of the monopole-string.

We write the 5D gauge field as

$$A = Q A^{EM} + \tilde{A} \quad , \tag{4.2}$$

where $A^{EM}$ is a unit monopole-string background homogeneous along $z$ with $A_z^{EM} = 0$, and $\tilde{A}$ encodes any smooth dynamics of normalizable modes including pions $\mathrm{U}(x)$, as well as additional smooth background potential $A_L$ and $A_R$ coupled to the chiral currents. Being a unit magnetic source, $A^{EM}$ is characterized by

$$-2\pi i = \int_{S^2} F^{EM} = \int_{S^2} dA^{EM} = \int_{B^3} d^2 A^{EM} \quad , \tag{4.3}$$

where in the last equality we use the Stokes theorem on the 3-ball $B^3$ around the monopole core. This gives us

$$d^2 A^{EM} = -2\pi i \, \delta_3(\vec{0}) \quad , \tag{4.4}$$

where $\delta_3(\vec{0})$ is a delta 3-form localized in space $\vec{x}$ at the monopole center $\vec{x} = \vec{0}$. In components $\delta_3(\vec{0}) = \delta^{(3)}(\vec{x}) \, dx^1 \wedge dx^2 \wedge dx^3$. With this gadget, it is straightforward to find the Bianchi identity violation in 5D,

$$DF \equiv dF + A \wedge F - F \wedge A = Q \, d^2 A^{EM} = -2\pi i \, Q \, \delta_3(\vec{0}) \quad , \tag{4.5}$$

so that $\mathrm{Tr}\,(F \wedge F)$ is no longer closed,

$$d\,\mathrm{Tr}\,(F \wedge F) = 2\mathrm{Tr}\,(DF \wedge F) = -4\pi i \, \mathrm{Tr}\,(QF) \wedge \delta_3(\vec{0}) \quad . \tag{4.6}$$

In components this is equivalent to

$$\frac{1}{4} \epsilon^{MNPQR} \partial_M \mathrm{Tr}\,(F_{NP} F_{QR}) = 4\pi i \delta^{(3)}(\vec{x}) \mathrm{Tr}\,(QF_{tz}) \quad , \tag{4.7}$$

which implies that

$$\partial_\mu \left( \epsilon^{\mu\nu\alpha\beta} \text{Tr} \left( F_{\nu\alpha} F_{\beta z} \right) \right) = -\frac{1}{4} \partial_z \left( \epsilon^{\mu\nu\alpha\beta} \text{Tr} \left( F_{\mu\nu} F_{\alpha\beta} \right) \right) + 4\pi i \delta^{(3)}(\vec{x}) \text{Tr} \left( Q F_{tz} \right) \quad . \tag{4.8}$$

This is precisely what we need to find the violation of baryon number (3.9),

$$\partial_\mu B^\mu = \frac{1}{8\pi^2} \int_{-\infty}^{+\infty} dz \, \partial_\mu \left( \epsilon^{\mu\nu\alpha\beta} \text{Tr} \left( F_{\nu\alpha} F_{\beta z} \right) \right) \tag{4.9}$$

$$= \frac{1}{32\pi^2} \left( \epsilon^{\mu\nu\alpha\beta} \text{Tr} \left( F_{\mu\nu} F_{\alpha\beta} \right) \Big|_R - \epsilon^{\mu\nu\alpha\beta} \text{Tr} \left( F_{\mu\nu} F_{\alpha\beta} \right) \Big|_L \right) + \frac{i\delta^{(3)}(\vec{x})}{2\pi} \int_{-\infty}^{+\infty} dz \, \text{Tr} \left( Q F_{tz} \right),$$

where we will ignore the first term as we already discuss it in the previous section.

To study the monopole-induced second term, let us go back to the $A_z = 0$ gauge and expand $A_\mu(x, z)$ more precisely,

$$A_\mu = \left[ \left( U^{-1}(Q A_\mu^{EM} + A_{L\mu}) U \right) \psi_+ + (Q A_\mu^{EM} + A_{R\mu}) \psi_- \right] + \psi_+ U^{-1} \partial_\mu U + \sum_{k \geq 1} B_\mu^{(k)} \psi_k \quad , \tag{4.10}$$

including now the complete spectrum of excited (axial) vector mesons in the expansion. Since $A_t^{EM} = 0$, and $F_{tz} = -\partial_z A_t$ in our gauge, the $z$-integral is readily performed to give

$$\partial_\mu B^\mu = -\frac{i\delta^{(3)}(\vec{x})}{2\pi} \text{Tr} \left( Q A_t \right) \Big|_{-\infty}^{+\infty}$$

$$= -\frac{i\delta^{(3)}(\vec{x})}{2\pi} \left[ \text{Tr} \left( Q U^{-1} \partial_t U \right) + \text{Tr} \left( Q U^{-1} A_{Lt} U \right) - \text{Tr} \left( Q A_{Rt} \right) \right] \quad . \tag{4.11}$$

This is the main result in this section. Note that there is no contribution from excited (axial) vector mesons because their wavefunctions $\psi_k(z)$ vanish sufficiently fast near the boundaries. It is easy to see that the first term precisely reproduces to the 4D result (2.7). Writing $U(x) = \exp(\frac{2i}{F_\pi} \pi^0(x) \sigma^3)$ as before, we have

$$\partial_\mu B^\mu = -\frac{i\delta^{(3)}(\vec{x})}{2\pi} \text{Tr} \left( Q \sigma^3 \right) \frac{2i(\partial_t \pi^0)}{F_\pi} = \frac{(\partial_t \pi^0)}{\pi F_\pi} \delta^{(3)}(\vec{x}) \quad , \tag{4.12}$$

whose $\vec{x}$-space integration is nothing but (2.7).

It is also interesting to speculate the implication of the second and the third terms. They seem to indicate that in the presence of a chiral asymmetric chemical potential, monopoles can create/annihilate baryon number. It would be interesting to understand this better.

## 5. String theory realization

There is a nice stringy set-up realizing the physics of the previous sections in terms of $D$- branes in the Sakai-Sugimoto model. Let us parameterize the cigar-shaped part of the gravity background by a radial coordinate $U \geq U_{KK}$ and an angle $\tau \sim \tau + 2\pi M_{KK}^{-1}$ [2]. Details of the parameters $U_{KK}$ and $M_{KK}$ are not relevant in our discussion. Our $N_F$ probe $D8$ branes are spanning a line $\{\tau = 0\} \cup \{\tau = \pi M_{KK}^{-1}\}$, $U \geq U_{KK}$ in the cigar part.

They also wrap the internal $S^4$ fibration and span the Minkowski space $R^{1,3}$. We then consider a $D6$ brane which wraps the internal $S^4$ fibration and spans a half of the cigar $0 \leq \tau \leq \pi M_{KK}^{-1}$, $U \geq U_{KK}$, ending on one of the $N_F$ $D8$ branes. It is point-like in the spatial $\vec{x}$ and static along the time.

Ignoring $S^4$ and $U$ directions since they are common to the $D8$ and $D6$ branes, the system is similar to a $D1$ brane ending perpendicularly on one of $N_F$ $D3$ branes. The end point of $D1$ on $D3$ looks like a monopole source in view of $D3$ world volume gauge field. Because only one end of the $D1$ ($D6$) brane is on the $D3$ ($D8$) brane while the other end extends to infinity, the resulting monopole configuration on the $D3$ ($D8$) brane world volume is an Abelian Dirac monopole with infinite energy in the spatial $\vec{x}$ directions. Including the radial direction $U$ in the 5D $D8$ brane world volume (we ignore $S^4$), or the $z$ direction in the previous sections, what we have is precisely a 5D holographic, Dirac monopole-string in the 5D gauge theory on the $D8$ branes. Its $\mathrm{U}(1)$ monopole charge direction depends on which $D8$ brane the $D6$ brane ends, and we simply call it the charge matrix $Q$. For an example of $N_F = 2$ with the $D6$ brane ending on the first $D8$ brane, we have

$$Q = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad . \tag{5.1}$$

As the monopole-string is homogeneous along $U$ (or $z$), it represents a vector-like background gauge potential of the chiral symmetry (or $A_L = A_R = Q A^{Dirac}$) with a monopole charge $Q$ in holographic QCD. Therefore, the physics of monopole catalysis of baryon decay that we study in the previous sections must apply to this stringy setting.

Indeed, we can easily identify the string theory phenomenon corresponding to the baryon decay in the presence of a $D6$ monopole-string. 5D instanton-baryons on the $D8$ branes can be thought of as $S^4$-wrapped $D4$ branes dissolved into the $D8$ branes. But, these $S^4$-wrapped $D4$ branes can also dissolve into our $D6$ brane, because they are similar to a $D0/D2$ system when we ignore the common $S^4$ directions. Therefore, $D4$-baryons can be captured by the $D6$ monopole-string and disappear from the $D8$ world volume. This is the string theory correspondent to the monopole catalysis of baryon decay.

We define the baryon number of a single $S^4$-wrapped $D4$ brane to be one, as it also has unit instanton number on the $D8$ branes. The dissolved $D4$-baryon number into our $D6$ monopole-string is measured by the 2-form field strength $F^{(2)} = dA^{D6}$ of the $D6$ world volume gauge potential on the half cigar that the $D6$ world volume spans, similar to $D0/D2$ system,

$$(\Delta B)_{D6} = \frac{i}{2\pi} \int_{(U,\tau)} F^{(2)} = -\frac{i}{2\pi} \int_{-\infty}^{+\infty} dz\, A_z^{D6} \quad , \tag{5.2}$$

where the normalization can be fixed by that a unit $(-2\pi i)$ flux represents a single dissolved $D4$-brane, and we use the Stokes theorem in the last equality since the boundary of the $D6$ half cigar is precisely the line along $z$ at the $D6/D8$ intersection. Note that in $\vec{x}$ space, this is precisely the position of the monopole-string core. As the $D6$ brane ends on one of the $D8$ branes, the $D6$ world volume gauge potential is identical to the corresponding $D8$ world volume gauge potential at the $D6/D8$ intersection. Therefore, the above $A_z^{D6}$ can

be equally interpreted as the $A_z$ at the monopole-string core on the $D8$ brane on which the $D6$ brane ends,

$$A_z(t, \vec{0}) = Q A_z^{D6}(t) \quad , \tag{5.3}$$

where we take the monopole position at the origin $\vec{x} = \vec{0}$, and $Q$ represents the charge matrix of $D6$ ending on the $D8$ brane, as given before. Using $\text{Tr}(Q^2) = 1$, we have

$$(\Delta B)_{D6} = -\frac{i}{2\pi} \int_{-\infty}^{+\infty} dz \, \text{Tr} \left( Q A_z \right) \Big|_{\vec{x}=\vec{0}} \quad , \tag{5.4}$$

whose time derivative must be equal to the rate of disappearance of the baryon number from the $D8$ branes,

$$\frac{dB}{dt} = -\frac{d}{dt}(\Delta B)_{D6} = -\frac{i}{2\pi}\frac{d}{dt}\text{Tr}\left( -Q \int_{-\infty}^{+\infty} dz \, A_z \right) \quad . \tag{5.5}$$

Noting that the integral inside the trace is precisely the Wilson line corresponding to the pions,

$$-\int_{-\infty}^{+\infty} dz \, A_z = \frac{2i}{F_\pi} \pi^0(\vec{0}) \sigma^3 \quad , \tag{5.6}$$

we finally have

$$\frac{dB}{dt} = \frac{1}{\pi F_\pi} \text{Tr} \left( Q \sigma^3 \right) \left( \partial_t \pi^0 \right) \quad . \tag{5.7}$$

This is precisely what we have in the previous sections.


# 6. Conclusion

Fermion number is often not conserved in background fields which modify the spectrum of fermions [18]. One tantalizing such phenomenon is the monopole catalysis of baryon decay, where baryons disappear (or appear) near the magnetic monopole. Since monopole catalysis of baryon decay may have a significant effect in monopole search and proton decay experiment, both of which are consequences of unified gauge theories, it is desirable to understand it more clearly. We have investigated monopole catalysis in the context of the gauge/string duality and showed how it is realized in holographic QCD and also in string theory. In doing so we have demonstrated that the gauge/string duality is indeed a powerful tool to study strong interactions, and found that the baryon number violation under the magnetic monopole or by the electroweak sphaleron can be formulated into a single equation in holographic QCD.

There are several phenomenological implications of our study. One of them is the generation of baryons in the presence of magnetic monopole by external chiral chemical potentials, as shown in eq. (4.11), which might be more effective in generating baryon asymmetry at lower temperature where the sphalerons are suppressed. This mechanism might be relevant in early universe or heavy ion collision but we leave it for future investigation.

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